Book Notes 2023-04-03 P. Cyrenne - Work in Progress – Not to be Cited

Chapter 5: The Optimal Use of Fossil Fuels

3.1 **Introduction**

As is well known almost all governments are focused on the issue of climate change. In doing so, they are basing their policies on the relationship between greenhouse gas emissions and changes to the climate. A particular greenhouse gas of concern is carbon dioxide (CO_2). Governments focus on the release of carbon dioxide which occurs with the burning of fossil fuels, specifically, oil, natural gas, diesel, and gasoline in particular. Most governments are then concerned with limiting carbon dioxide emissions either using market type mechanisms, carbon taxes, tradeable permits, or command and control mechanisms which involves setting an outright limit on emissions.

In thinking about the role of CO_2 emissions and climate change, it is important for modelling purposes to distinguish between global warming, and increase in average recorded temperature, and climate change in general. The latter might be described by an increase in the variability of the average temperature.

In this note, we focus on effect of increases in energy use, particularly the use of energy derived from fossil fuels, and the effects of the subsequent carbon dioxide emissions CO_2 and the average temperature, but will discuss the issue of climate variability at the end of this chapter.

In doing so, I incorporate a number of features that are included in large scale models that look at the relationship between fossil fuel use, the environment and the economy. Using a simplified model, I think a number of key issues can be easily illustrated which can clarify the transmission mechanism of fossil fuel use and economic and environmental effects. It is felt that this can be valuable both for students and researchers as well as the public.

I find as has been emphasized that the issue of the environmentally sustainable energy use depends on a number of a key parameters with many of these parameters only known with any precision in future periods. In addition, future technological change has also been emphasized as playing a key role in offsetting some of the deleterious effects of fossil fuels used today.

Most discussions of climate change center around the issue of the emissions of carbon dioxide as a consequence of fossil fuel use. While large scale climate models capture the links between energy use, economic growth and environmental effects, the links between these elements is often opaque. This note examines the optimal use of fossil fuels in relation to the emissions of carbon dioxide, economic growth, wealth effects and changes in the average temperature in a simple two period model. I show that many of the policy disputes regarding climate policy are easily discussed and relate to disagreements regarding the size of the parameters in the model.

3.2 A Model of Fossil Fuel Use and CO_2 emissions and Welfare

To model the link between CO_2 emissions and consumer welfare, in is important to be clear regarding the transmission mechanism that relates fossil fuel use defined as E to CO_2 emissions and hence climate effects. Two main alternatives present themselves. The first hypothesis is that greater fossil fuel use leads to greater CO_2 emissions which then affects the average temperature of the planet and subsequently consumer welfare both in terms of current income as well as the effects of the stock of CO_2 on wealth. This might be thought of as the Global Warming hypothesis of welfare change. The changes in income and wealth are particularly true of sectors of the economy which are sensitive to changes in average atmospheric temperature. One sector is agriculture.[i]

The second alternative is that increasing fossil fuel use leads to increased concentrations of CO_2 in the atmosphere which leads to more variation in the average temperature. This can be called the Climate Change hypothesis. It is argued the larger variation in average temperature can have a deleterious effect on incomes and the stock of wealth if it leads to more extreme weather events.[ii]

The optimization problem is to determine the optimal fossil fuel use and hence CO_2 emissions allowed for each period, taking into account the effect of the level of CO_2 on GDP and average temperature in each period. In what follows the subscript on CO refers to the CO_2 emissions in the respective period.

$$Z = GDP_1 + \beta GDP_2 + \beta W_2 - p_1E_1 - p_2E_2$$

Welfare is assumed to be the second period welfare level which is being discounted $0 < \beta \leq 1$. With β<1 implying that future welfare is discounted in relation to the present period.

3.3 The Optimal Use of Fossil Fuels: A Specific Functional Form

In this section, we consider a specific functional form version of the two period model introduced in Section II. The objective function allows for GDP levels in each of the two periods. It also includes the stock wealth in the second period. This wealth is assumed to be effected by the average temperature which will result in the second period. For example, as the average temperature increases it is suggested that wealth in the form of say coastal property in the future is likely to be adversely effected.

$$Z = GDP_1 + \beta GDP_2 + \beta W_2 - p_1E_1 - p_2E_2$$

$$GDP_1 = \alpha_1E_1 - E_1^2 \tag{6}$$

$$GDP_2 = \alpha_2E_2 - E_2^2 \tag{7}$$

$$CO_1 = e_1 E_1 \tag{8}$$

$$CO_2 = e_2 E_2 + \delta CO_1 \tag{9}$$

$$T_2 = \theta\left(CO_2 + CO_1 - \overline{CO}\right) \tag{10}$$

$$W_2 = \overline{W} - \gamma T_2 \tag{11}$$

For what follows we drop the subscript that defines carbon dioxide and just write CO where CO_1 (CO_2) are the respective levels of carbon dioxide emitted in the first (second) period.

Equation (6) specifies that energy used in the first period E_1 affects the GDP in the first period, given α_1 while (7) is the effect of energy used in the second period E_2 on GDP in the second period. I assume that the marginal effect of fossil fuel use on GDP in each period is diminishing.

Equation (8) suggests that carbon dioxide emitted in the first period CO_1 is some fraction of energy use E_1 in that period, while (9) allows for the fact that the some CO_1 emitted in the first period remains in the atmosphere and contribute to the total stock of carbon dioxide in the second period with $0 < \delta < 1$.[iii] Equation (10) outlines the relationship between emissions of carbon dioxide and average temperature in the second period given by the respective parameter θ. Because of the persistence of carbon dioxide in the atmosphere, the average temperature in the second period is partly determined by the emissions of carbon dioxide in the first period (CO_1).

Equation (11) captures the effect of the increase in the average temperature in period two T_2 on the stock of wealth in the second period with the respective parameter γ capturing the strength of that effect. It is possible that the model includes an increase in average temperature on first period wealth, but I think the second period effects are of greater interest, since in general, the concern is how present period energy use affects future periods.

Substituting (8), and (9) into (10) yields

$$T_2 = \theta(e_2 E_2 + \delta e_1 E_1 - \overline{CO}) \tag{12}$$

And then into (11) yields

$$W_2 = \overline{W} - \gamma\theta\left(e_2 E_2 + \delta e_1 E_1 - \overline{CO}\right) \tag{13}$$

Case 1: Linear Temperature Effects from Carbon Dioxide Emissions

As outlined by equation (11) indicates that the average temperature in period two is increasing in the stock of CO_2 given the residual stock of CO_2 from the first period (δCO_1). The objective function can now be rewritten as

$$Z = (\alpha_1 E_1 - E_1^2) + \beta\left(\alpha_2 E_2 - E_2^2\right) + \beta[(\overline{W -} \gamma\theta\left(e_2 E_2 + \delta e_1 E_1 - \overline{CO}\right)] - p_1 E_1 - p_2 E_2$$

We assume that a central planner chooses the level of fossil fuel use in each of the two periods, E_1 and E_2 to maximize welfare which is defined over the two periods.

$$E_1: \ \alpha_1 - 2E_1 - \ \beta\gamma\theta\delta e_1 - p_1 = 0$$

$$E_2: \quad \beta(\alpha_2 - 2E_2) \ - \beta\gamma\theta \ e_2 \ - p_2 = 0$$

Collecting terms in E_1 and E_2

$$E_1^* = \frac{(\alpha_1 - p_1 - \beta\gamma\theta\delta e_1)}{2} \tag{14}$$

$$E_2^* = \frac{(\beta\alpha_2 - p_2 - \beta\gamma\theta e_2)}{2\beta} \tag{15}$$

Proposition 1. (Linear Average Temperature Case) The fossil fuel use in the first period E_1^* is (i) increasing in the importance of energy in increasing GDP_1 in the first period (α_1) but is decreasing in all other parameters in the model, the fossil fuel price in the first period p_1, the discount rate β, carbon emissions e_1, the extent of carbon dioxide persistence in the atmosphere δ, θ the extent to which carbon dioxide increases average temperatures and γ the extent to which the stock of wealth is adversely affected by increases in average temperatures. Corresponding results hold for fossil fuel use in the second period E_2^*. Proof: Simple comparative statics.

Most of these results accord with intuition. If we focus on second period energy use, the parameters capture the policy debates regarding energy use quite clearly. Optimists suggest that future GDP may not be generate as much CO_2, (e_2 will be lower) , that the effect of CO_2 on average temperatures is overestimated (θ is lower) and second period wealth will not be as adversely effected by rising average temperatures (γ is lower).

It is interesting to note that in this specification, the critical level of carbon dioxide CO_2 does not directly affect the energy use in either period. The next case outlines that possibility.

Case 2: Increasing Average Temperature from Carbon Dioxide Emissions

Here to simplify the algebra I set γ= 1 and $e_1 = e_2 = 1$, and using (12) with the amendment that marginal reduction in wealth in the second period is adversely effected by increasing average temperature from carbon dioxide emissions.

$$W_2 = \overline{W_2} - \theta\left(E_2 + \delta E_1 - \overline{CO}\right)^2$$

The objective function can now be rewritten as

$$Z = (\alpha_1 E_1 - E_1^2) + \beta\left(\alpha_2 E_2 - E_2^2\right) + \beta[\overline{W_2} - \theta\left(E_2 + \delta E_1 - \overline{CO}\right)^2] - p_1 E_1 - p_2 E_2$$

$$E_1\colon \alpha_1 - 2E_1 - 2\beta\theta\delta\left(E_2 + \delta E_1 - \overline{CO}\right) - p_1 = 0$$

$$E_2\colon\ \beta(\alpha_2 - 2E_2) - 2\beta\theta(E_2 + \delta E_1 - \overline{CO}) - p_2 = 0$$

$$E_1^* = \frac{\left[(1+\theta)(\alpha_1 - p_1) - \theta\delta(\beta\alpha_2 - p_2) + 2\beta\theta\delta\overline{CO}\right]}{2(1+\theta+\delta^2)} \tag{16}$$

$$E_2^* = \frac{\left[(1+\beta\theta\delta^2)(\beta\alpha_2 - p_2) - \beta\theta(\alpha_1 - p_1) + 2\beta\theta\overline{CO}\right]}{2(1+\theta+\delta^2)} \tag{17}$$

Proposition 2. (Increasing Average Temperature Case). Focusing on the second period energy use, E_2^* is increasing in (i) the second period discounted market size ($\beta\alpha_2$) (ii) the critical level of carbon dioxide ($\overline{CO}$) and (iii) is decreasing in first period market size (α_1). Proof: Simple comparative statics.

Each of these effects is influenced by a number of key parameters in the model, β, θ, δ. Regarding second period fossil fuel use E_2^*, of interest it is decreasing in the difference between the market size parameter and the first period price of energy ($\alpha_1 - p_1$). This effect is stronger the lower is the discount rate (higher β) and the lower is the temperature increase from the total amount of carbon dioxide in the atmosphere (lower θ).

We can examine a couple of special cases.

(i) First suppose $\theta = 1$ then

$$E_1^* = \frac{\left[2(\alpha_1 - p_1) - \delta(\beta\alpha_2 - p_2) + 2\beta\delta\overline{CO}\right]}{2(2+\delta^2)} = \frac{\alpha_1 - p_1}{(2+\delta^2)} - \frac{\delta(\beta\alpha_2 - p_2)}{2(2+\delta^2)} + \frac{\beta\delta\overline{CO}}{(2+\delta^2)}$$

$$E_2^* = \frac{\left[(1+\beta\delta^2)(\beta\alpha_2 - p_2) - \beta(\alpha_1 - p_1) + 2\beta\overline{CO}\right]}{2(2+\delta^2)} = \frac{(1+\beta\delta^2)(\beta\alpha_2 - p_2)}{2(2+\delta^2)} - \frac{\beta(\alpha_1 - p_1)}{2(2+\delta^2)} + \frac{\beta\overline{CO}}{(2+\delta^2)}$$

(ii) Second, suppose in addition $\delta = 1$, which means all carbon dioxide emissions in the first period CO_1 remain in the atmosphere in the second period.

$$E_1^* = \frac{\left[2(\alpha_1 - p_1) - (\beta\alpha_2 - p_2) + 2\beta\overline{CO}\right]}{6}$$

$$E_2^* = \frac{\left[(1+\beta)(\beta\alpha_2 - p_2) - \beta(\alpha_1 - p_1) + 2\beta\overline{CO}\right]}{6}$$

If $\beta_1 = 1$, that is there is no discounting and $(\alpha_1 - p_1) = (\alpha_2 - p_2) = (\alpha - p)$ the above simplifies to

$$E_1^* = \frac{\left[(\alpha - p) + 2\overline{CO}\right]}{6} = E_2^*$$

3.4 **Carbon Emissions Budget**

Regarding the effect of CO_2 emissions, in this section I consider a two period carbon emissions budget that is binding. The planner is assumed to choose E_1 and E_2 to maximize

$$Z = GDP_1 + \beta GDP_2 + \beta W_2 - p_1 E_1 - p_2 E_2$$

Subject to the carbon emissions constraint $\overline{CO} = CO_1 + CO_2$. Substituting (6) to (9) and assuming $T_2 = \theta(e_2E_2 + \delta e_1E_1)$ and $W_2 = \overline{W_2} - \gamma T_2$ or $W_2 = \overline{W_2} - \gamma\theta(e_2E_2 + \delta e_1E_1)$ and taking the first order conditions with respect to E_1 and E_2 subject to the budget constraint yields

$$£ = (\alpha_1E_1 - E_1^2) + \beta(\alpha_2E_2 - E_2^2) + \beta[(\overline{W_2} - \gamma\theta(e_2E_2 + \delta e_1E_1)] - p_1E_1 - p_2E_2 + \lambda(\overline{CO} - e_2E_2 - (1+\delta)e_1E_1)$$

$$E_1\colon \alpha_1 - 2E_1 - \beta\gamma\delta e_1E_1 - p_1 - \lambda(1+\delta)e_1 = 0$$

$$E_2\colon \beta(\alpha_2 - 2E_2) - \beta\gamma e_2E_2 - p_2 - \lambda e_2 = 0$$

$$\lambda\colon \overline{CO} - e_2E_2 - (1+\delta)e_1E_1 = 0$$

Where $\theta = 1$ and $e_1 = e_2 = 1$ for algebraic simplicity, yields the determinant of this system is $D = (2 + \beta\gamma\delta) + \beta(1+\delta)^2(2+\gamma)$. Given these restrictions, solving for E_1, E_2 and λ, yields

$$E_1^* = \frac{\left[(\alpha_1 - p_1) - (1+\delta)(\beta\alpha_2 - p_2) + \beta(2+\gamma)\overline{CO}\right]}{D}$$

$$E_2^* = \frac{\left[(1+\delta)^2(\beta\alpha_2 - p_2) - (1+\delta)(\alpha_1 - p_1) + (2+\beta\gamma)\overline{CO}\right]}{D}$$

$$\lambda^* = \frac{\left[(2+\beta\gamma\delta)(\beta\alpha_2 - p_2) + (1+\delta)\beta(2+\gamma)(\alpha_1 - p_1) - \beta(2+\gamma)(2+\beta\gamma\delta)\overline{CO}\right]}{D}$$

Proposition 3. (Binding Carbon Emissions Constraint) The optimal use of fossil fuels in the first period E_1^* is (i) increasing in the difference between first period market size and first period price $(\alpha_1 - p_1)$, and is (ii) decreasing in the difference between second period market size and second period price $(\beta\alpha_2 - p_2)$. Second period fossil fuel has the opposite relationship to the respective differences in market sizes over price. Fossil fuel use in both periods are (iii) increasing in the critical carbon dioxide level $\overline{CO}$ after which carbon dioxide emissions raise average temperature. Finally the shadow price of the carbon dioxide emissions λ^* in the atmosphere is (iv) increasing in the respective market sizes, (v) decreasing in fossil fuel prices in each period and (vi) is decreasing in the critical carbon dioxide level $\overline{CO}$ after which average temperature T rises and second period wealth W_2 is adversely effected.

3.5 Discussion and Conclusions

This note has highlighted a number of issues related to the use of fossil fuels in a simple two period model. It has captured a number of key features in the debate which can allow for a fruitful discussion of the issues involved regarding fossil fuel use over time in relation to carbon emissions and the climate.

A key finding is that much of the debate centres around the trend of key parameters in the model. For example, will technological change result in future carbon dioxide emissions in relation to the GDP of economies. Second, to what extent will the buildup of the stock of carbon dioxide contribute to the rise in average temperature in the future. Third, how likely is the future stock of wealth likely to be adversely effected by either a rise in average temperatures or an increase in the variability of average temperature, depending on whether global warming or climate change is the more serious threat.

3.6 References

Keith, David W. (2009) “Why Capture CO_2 from the Atmosphere?” *Science*, 235:1654-1655.

Hansen, James and Pushker Kharecha (2018) “Cost of Carbon Capture: Can Young People Bear the Burden?” *Joule*, 2, 1405-1407.

Endnotes

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[i] It should be noted that even in this case, it is possible that in cold countries like Canada and Northern Europe, higher average temperatures may increase crop yields and hence agricultural production.

[ii] It is clear that increasing use of fossil fuels and hence increasing concentrations of CO_2 can contribute to both higher average temperatures (global warming) and greater variability of average temperature (climate change). In this note, we model the two hypotheses separately.

[iii] It has been suggested that the impact of carbon emissions persists longer than that of nuclear waste, as reported by Keith (2009:1654).